Coimisiún na Scrúduithe Stáit

State Examinations Commission

Leaving Certificate 2015

Marking Scheme

Applied Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

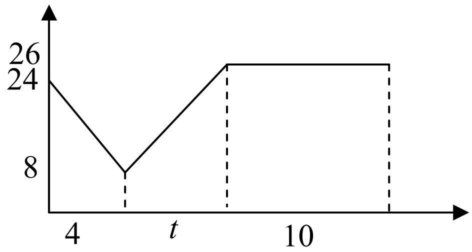
Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2 The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. The points P and Q lie on a straight level road.
A car passes P with a speed of 24 m s^{-1} and decelerates uniformly for 4 seconds to a speed of 8 m s^{-1}.
The car now accelerates uniformly from 8 m s^{-1} to a speed of 26 m s^{-1}.
The car travels 102 metres while accelerating.
It now continues at a constant speed of 26 m s^{-1} for 10 seconds and then passes Q.

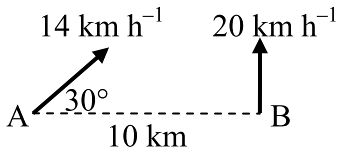


(a) Find **(i)** the deceleration
(ii) the acceleration
(iii) $|PQ|$, the distance from P to Q
(iv) the average speed of the car between P and Q.

(b) There is a legal speed limit of 100 km h^{-1} on this road.
Investigate if the car exceeds the speed limit as it travels from P to Q.

(a)	(i)	$v = u + at$	
		$8 = 24 + a(4)$	
		$a = -4 \text{ m s}^{-2}$	10
	(ii)	$v^2 = u^2 + 2as$	
		$26^2 = 8^2 + 2a(102)$	
		$a = 3 \text{ m s}^{-2}$	10
	(iii)	$s = ut + \frac{1}{2}at^2$	
		$s_1 = 24(4) + \frac{1}{2}(-4)(16) = 64 \text{ m}$	
		$\lvert PQ\rvert = 64 + 102 + 26(10) = 426 \text{ m}$	10
	(iv)	$v = u + at$	
		$26 = 8 + 3(t) \Rightarrow t = 6 \text{ s}$	
		$v_1 = \dfrac{426}{4+6+10} = 21.3 \text{ m s}^{-1}$	10
(b)		$100 \text{ km h}^{-1} = 100 \times \dfrac{5}{18} = 27.\dot{7} \text{ m s}^{-1}$	5
		$26 < 27.\dot{7}$	5
			50

2. Ship A is travelling east 30° north at a constant speed of 14 km h^{-1}.



Ship B is travelling due north at a constant speed of 20 km h^{-1}.
B is positioned 10 km due east of A.

(i) Express the velocity of A and the velocity of B in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

(iii) Calculate the shortest distance between the ships in their subsequent motion.

(iv) Find the distance between the ships one hour after the instant that they were closest together.

(i)

$$\vec{V}_A = 14\cos 30\,\vec{i} + 14\sin 30\,\vec{j}$$

$$= 7\sqrt{3}\,\vec{i} + 7\,\vec{j} \qquad 10$$

$$\vec{V}_B = 0\,\vec{i} + 20\,\vec{j} \qquad 10$$

(ii)

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B \qquad 5$$

$$= 7\sqrt{3}\,\vec{i} - 13\,\vec{j} \qquad 5$$

(iii)

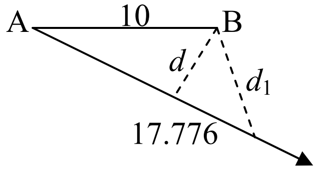
$$\alpha = \tan^{-1}\left(\frac{13}{7\sqrt{3}}\right) = 47.0° \qquad 5$$

$$d = 10\sin\alpha = 7.3 \text{ km} \qquad 5$$

(iv)

$$|V_{AB}| = \sqrt{(7\sqrt{3})^2 + 13^2}$$

$$= 17.776 \text{ km h}^{-1} \qquad 5$$



$$d_1 = \sqrt{7.3^2 + 17.776^2}$$

$$= 19.2 \text{ km} \qquad 5$$

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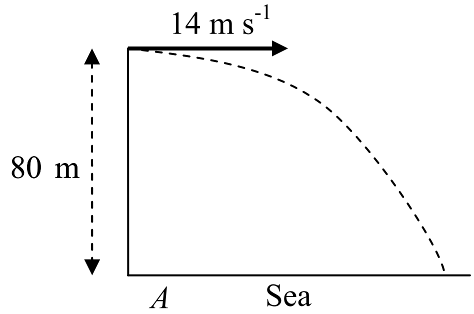
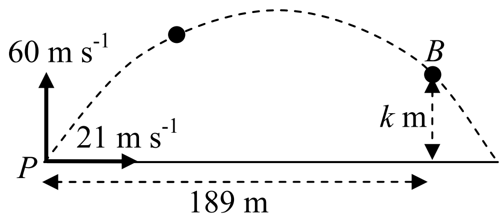
3. **(a)** A particle is projected horizontally with an initial speed of 14 m s^{-1} from the top of a straight vertical cliff of height 80 m.

How far from the foot of the cliff will it hit the sea?



(b) A particle is projected with initial velocity $21\vec{i}+60\vec{j}$ m s^{-1} from point P on a horizontal plane.
A and B are two points on the trajectory (path) of the particle.
The particle reaches point A after 4 seconds of motion.
The displacement of point B from P is $189\vec{i}+k\vec{j}$ m.



Find **(i)** the velocity of the particle at A in terms of $\vec{i}$ and $\vec{j}$
(ii) the speed and direction of the particle at A
(iii) the value of k.

(a)	$80 = 0 + 5\times t^2$	
	$t = 4$ s	10
	$d = 14\times 4 = 56$ m	10
(b)(i)	$v_y = u + at$	
	$= 60 - 10\times 4 = 20$	5
	$\vec{v} = 21\vec{\mathrm{i}} + 20\vec{\mathrm{j}}$	5
(ii)	$\lvert\vec{v}\rvert = \sqrt{21^2 + 20^2}$	
	$= 29$ m s^{-1}	5
	$\beta = \tan^{-1}\dfrac{20}{21} = 43.6^\circ$	5
(iii)	$21t = 189$	
	$t = 9$	5
	$s_y = ut + \frac{1}{2}at^2$	
	$k = 60\times 9 - 5\times 9^2$	
	$= 135$	5

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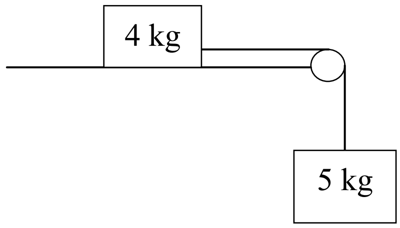
4. **(a)** A particle of mass 4 kg is connected to another particle of mass 5 kg by a taut light inelastic string which passes over a light smooth pulley at the edge of a rough horizontal table.

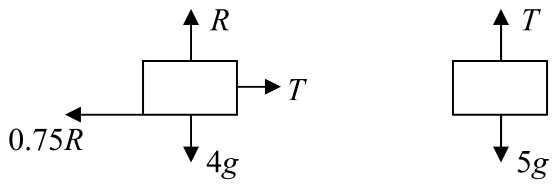


The coefficient of friction between the 4 kg mass and the table is $\frac{3}{4}$.

The system is released from rest.

(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the particles.
(iii) Find the tension in the string.

(i)



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(ii)

$$5g - T = 5a$$ 5

$$T - 0.75R = 4a$$ 5

$$5g - 0.75R = 9a$$

$$5g - 0.75(4g) = 9a$$

$$a = \frac{2g}{9} = 2.22 \text{ m s}^{-2}$$ 5

(iii)

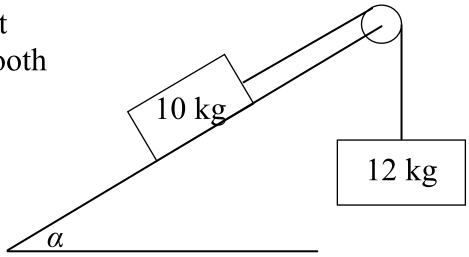
$$T = 5g - 5a$$

$$= 50 - 11.1$$

$$= 38.9 \text{ N}$$ 5

30

(b) Masses of 10 kg and 12 kg are connected by a taut light inelastic string which passes over a light smooth pulley, as shown in the diagram.



The 10 kg mass lies on a smooth plane inclined at α to the horizontal, where $\tan \alpha = \frac{4}{3}$.

The 12 kg mass hangs vertically.
The system is released from rest.

Find **(i)** the common acceleration of the particles
(ii) the tension in the string.

(i)

$$T - 10g\sin\alpha = 10a$$ 5

$$12g - T = 12a$$ 5

$$12g - 10g\left(\frac{4}{5}\right) = 22a$$

$$a = \frac{2g}{11} = 1.82\ \text{m s}^{-2}$$ 5

(ii)

$$T = 12g - 12a$$

$$= 98.2\ \text{N}$$ 5

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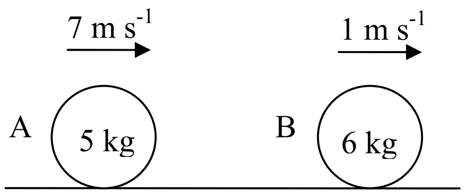
5. **(a)** A smooth sphere A, of mass 5 kg, collides directly with another smooth sphere B, of mass 6 kg, on a smooth horizontal table.



A and B are moving in the same direction with speeds of 7 m s^{-1} and 1 m s^{-1}, respectively.
The coefficient of restitution for the collision is $\frac{5}{6}$.

Find **(i)** the speed of A and the speed of B after the collision
(ii) the loss in kinetic energy due to the collision
(iii) the magnitude of the impulse imparted to A due to the collision.

(b) A ball is dropped from rest from a height of 3.2 m onto a smooth horizontal floor. The ball hits the floor and rebounds to a height of *h* metres above the floor.
The coefficient of restitution between the ball and the floor is $\frac{3}{5}$.

Find **(i)** the speed of the ball when it hits the floor
(ii) the value of *h*.

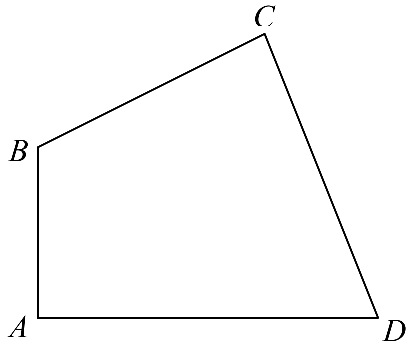
(a) (i)	$5(7) + 6(1) = 5v_1 + 6v_2$	5	
	$41 = 5v_1 + 6v_2$		
	$v_1 - v_2 = -\frac{5}{6}(7-1) = -5$	5	
	$v_1 = 1\ \text{m s}^{-1} \quad v_2 = 6\ \text{m s}^{-1}$	5	
(ii)	$\text{KE}_b = \frac{1}{2}(5)(7)^2 + \frac{1}{2}(6)(1)^2 = 125.5$		
	$\text{KE}_a = \frac{1}{2}(5)(1)^2 + \frac{1}{2}(6)(6)^2 = 110.5$		
	$\text{KE}_b - \text{KE}_a = 125.5 - 110.5 = 15\ \text{J}$	10	
(iii)	$\text{I} = \lvert(5)(1) - (5)(7)\rvert = 30$	5	
(b) (i)	$v^2 = 0 + 2\times 10\times 3.2$		
	$v = 8\ \text{m s}^{-1}$	10	
(ii)	$ev = \frac{3}{5}\times 8 = 4.8$	5	
	$0 = 4.8^2 + 2\times(-10)\times h$		
	$h = 1.152\ \text{m.}$	5	50

6. **(a)** Particles of weight 5 N, 9 N, 6 N and 1 N are placed at the points (p, 5), (7, q), (-6,-q) and (5, 8) respectively.
The co-ordinates of the centre of gravity of the system are (p, p).
Find **(i)** the value of p
(ii) the value of q.

(b) A quadrilateral lamina has vertices A, B, C and D.

The co-ordinates of the vertices are $A(0, 0)$, $B(0, 9)$, $C(12, 15)$ and $D(18, 0)$.

Find the co-ordinates of the centre of gravity of the lamina.



(a)

$$p = \frac{5(p)+9(7)+6(-6)+1(5)}{21}$$ 5

$$p = 2$$ 5

$$p = \frac{5(5)+9(q)+6(-q)+1(8)}{21}$$ 5

$$q = 3$$ 5

(b)

	area :	c.g.	
ABC	$\frac{1}{2}(9)(12) = 54$	$(4, 8)$	
ACD	$\frac{1}{2}(18)(15) = 135$	$(10, 5)$	5
lamina	$= 189$	(x, y)	5

$$(189)(x) = 54(4) + 135(10)$$
$$x = 8.3$$ 5

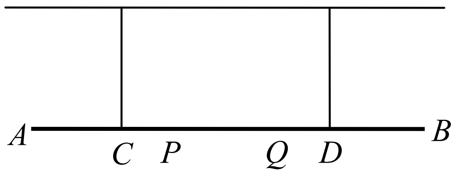
$$(189)(y) = 54(8) + 135(5)$$
$$y = 5.9$$

10

5

50

7. **(a)** A uniform beam, AB, is held in a horizontal position by two vertical inelastic strings attached at the points C and D respectively.

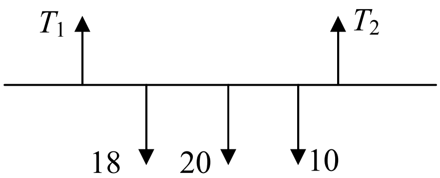


The weight of the beam is 20 N and the length of the beam is 4 m.

A particle of weight 18 N is placed at point P on the beam and another particle of weight 10 N is placed at point Q on the beam.

$|AC| = |BD| = 1$ m and $|CP| = |QD| = \frac{1}{2}$ m.

Calculate the tension in each of the strings.



$$T_1 + T_2 = 18 + 20 + 10$$

$$= 48$$

$$T_2 \times 2 = 18(0.5) + 20(1) + 10(1.5)$$

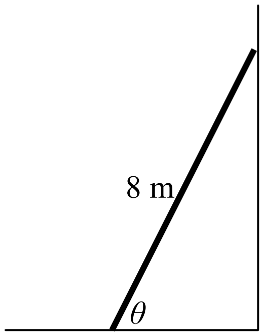
$$T_2 = 22 \text{ N}$$

$$T_1 = 26 \text{ N}$$

10	
10	
5	25

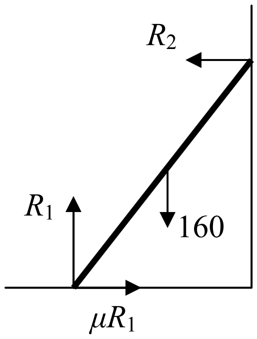
(b) A uniform ladder, of weight 160 N, is resting on rough horizontal ground and leaning against a smooth vertical wall.

The length of the ladder is 8 m.
The ladder makes an angle θ with the ground, where $\tan\theta = \frac{35}{12}$.



The ladder is in equilibrium and is on the point of slipping.

Find the coefficient of friction between the ladder and the ground.



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$$R_2 \times 8\sin\theta = 160 \times 4\cos\theta$$ 5

$$R_2 \times \tan\theta = 80$$

$$R_2 \times \frac{35}{12} = 80$$

$$R_2 = \frac{192}{7} = 27.4 \text{ N}$$ 5

$$R_1 = 160 \text{ N}$$ 5

$$\mu = \frac{R_2}{R_1} = 0.17$$ 5

25

8. **(a)** A particle describes a horizontal circle of radius 2 m with uniform angular velocity ω radians per second.
The particle completes 10 revolutions every minute.

Find **(i)** the value of ω
(ii) the speed and acceleration of the particle.

(i) $\dfrac{2\pi}{\omega} = 6$

$\omega = \dfrac{\pi}{3}\ \text{rad s}^{-1}$ 10

(ii) $v = r\omega$

$= 2\left(\dfrac{\pi}{3}\right)$

$= 2.1\ \text{m s}^{-1}$ 10

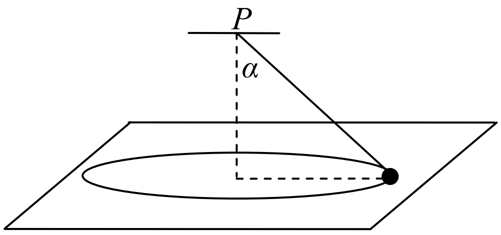
$a = r\omega^2$

$= 2\left(\dfrac{\pi}{3}\right)^2$

$= 2.2\ \text{m s}^{-2}$ 10 30

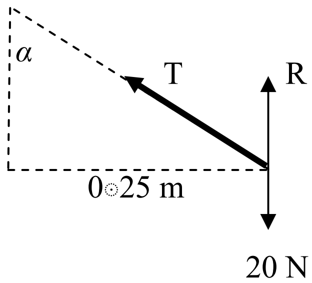
(b) A smooth particle of mass 2 kg is attached by a light inelastic string to a fixed point P.

The particle describes a horizontal circle of radius 0.25 m on the smooth surface of a horizontal table.



The centre of the circle is vertically below P.
The string makes an angle α with the vertical, where $\tan \alpha = \frac{4}{3}$.
The speed of the particle is 1.2 m s^{-1}.

Find **(i)** the tension in the string
(ii) the reaction force between the particle and the table.



(i)

$$T \sin \alpha = \frac{mv^2}{r}$$

$$T\left(\frac{4}{5}\right) = \frac{2(1.2)^2}{0.25}$$

$$T = 14.4 \text{ N}$$

10

(ii)

$$T \cos \alpha + R = 20$$

$$14.4 \times \frac{3}{5} + R = 20$$

$$8.64 + R = 20$$

$$R = 11.36 \text{ N}$$

10

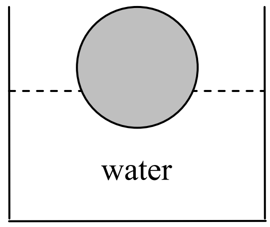
20

9. **(a)** A solid sphere floats at rest in water.

The radius of the sphere is 12 cm.

25% of the volume of the sphere lies below the surface of the water.

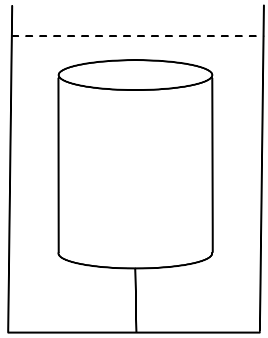
Find the weight of the sphere, correct to the nearest newton.
[Density of water = 1000 kg m^{-3}]



(b) A right circular solid cylinder has a height of 15 cm and a radius of 4 cm.

The relative density of the cylinder is 0.8 and it is completely immersed in a tank of liquid of relative density 1.2.

The cylinder is held at rest by a light inelastic vertical string which is attached to the base of the tank.



Find the tension in the string, correct to the nearest newton

(a)

$W = B$ — 5

$W = 1000 \times \frac{1}{4} \times \frac{4}{3}\pi\left(0.12^3\right) \times 10$

$W = 18$ N — 10

(b)

$B = 1200\{\pi \times (0{\cdot}04)^2(0.15)\}(10)$

$= 2.88\pi$ — 10

$W = 800\{\pi \times (0{\cdot}04)^2(0.15)\}(10)$

$= 1.92\pi$ — 10

$T + W = B$ — 5

$T = 2.88\pi - 1.92\pi$

$= 0.96\pi = 3$ N — 10 — 50

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